University of Gdańsk

Subject card

Subject name and code	Calculus on manifolds, PG_00170282		
Field of study	Mathematics		
Date of commencement of studies	October 2025	Academic year of realisation of subject	2025/2026
Education level	Master's studies	Subject group	Optional subject group
Mode of study	full-time studies	Mode of delivery	at the university
Year of study	1	Language of instruction	Polish Polish or English
Semester of study	2	ECTS credits	6.0
Learning profile	academic	Assessment form	exam
Conducting unit	Institute of Mathematics -> Faculty of Mathematics, Physics and Informatics -> Rector		
Name and surname of lecturer (lecturers)	Subject supervisor	dr hab. Andreas Zastrow	
	Teachers		

Lesson types

Lesson type	Lecture	Tutorial	Laboratory	Project	Seminar	SUM
Number of study hours	30.0	30.0	0.0	0.0	0.0	60

E-learning hours included: 0.0

Learning activity and number of study hours

Learning activity	Participation in didactic classes included in study plan	Participation in consultation hours	Self-study	SUM
Number of study hours	60	10.0	80.0	150

Subject objectives

To familiarize students with the main foundations of the theory of differential manifolds, basic concepts and tools, main theorems and proofs of some of them.

Developing students' ability to understand problems abstractly and use the theory of differential manifolds in mathematical scientific work

Learning outcomes

Course outcome	Subject outcome	Method of verification
[MATMU2_K04] is ready to understand and appreciate the importance of intellectual honesty in one's own and other people's actions; ethical conduct	The student is ready to understand and appreciate the importance of intellectual honesty in his own and other people's actions; ethical conduct	[SK1] oral statement/conversation/ discussion [SK8] observation of student's independent or team work
[MATMU2_K01] is willing to acknowledge the limitations of his or her own knowledge and is willing to pursue further education	The student is ready to acknowledge the limitations of one's own knowledge and pursue further education.	[SK1] oral statement/conversation/ discussion [SK8] observation of student's independent or team work
[MATMU2_U01] can construct mathematical reasoning: prove theorems and refute hypotheses through construction and selection of counterexamples	In the field of analysis on manifolds, the student is able to construct mathematical arguments: prove theorems and refute hypotheses through construction and selection of counterexamples.	[SU2] presentation/project/paper/ report [SU8] observation of student's independent or team work
[MATMU2_U06] is able to apply methods and examples from a selected field of mathematics in related fields	The student is able to solve problems in the field of analysis on manifolds, using as well standard methods of mathematical analysis, algebra and topology, as well as the definitions, properties and theorems learned during the lecture.	[SU1] oral statement/conversation/ discussion [SU8] observation of student's independent or team work
[MATMU2_U07] is able to define his/her interests and develop them; in particular, is able to establish contact with specialists in his/her field, e.g. understand their lectures intended for young mathematicians	The student is able to define his/ her interests and develop them; in particular, he is able to establish contact with specialists in the field of manifold analysis, e.g. understand their lectures intended for young mathematicians.	[SU1] oral statement/conversation/ discussion [SU8] observation of student's independent or team work
[MATMU2_K02] is ready to precisely formulate questions to deepen his/her understanding of a given topic or find missing elements of reasoning	In the field of analysis on manifold, the student is ready to precisely formulate questions to deepen his or her understanding of a given topic or find missing elements of reasoning.	[SK2] presentation/project/paper/ report [SK8] observation of student's independent or team work
[MATMU2_W02] knows and understands well the role and importance of the construction of mathematical reasoning	The student knows and understands well the role and importance of constructions of mathematical reasoning in the field of analysis on manifolds.	[SW1] oral statement/ conversation/discussion
[MATMU2_U03] can understand mathematical texts of various types from selected fields of mathematics	The student is able to understand mathematical texts of various types, in the field of analysis on manifolds.	[SU8] observation of student's independent or team work
[MATMU2_W03] knows and understands in-depth a selected field of theoretical or applied mathematics and is able to understand the formulations of issues in this field that are still at the research stage and knows the connections of issues in this field with other areas of mathematics	The student knows and understands analysis on manifolds in-depth and is able to understand the formulations of issues remaining at the research stage and knows the connections of issues with other areas of mathematics, in particular mathematical analysis, algebra or topology.	[SW4] test/exam - oral or written
[MATMU2_K05] is ready to independently search for information in literature, also in foreign languages	The student is ready to independently search for information in the literature in the fields of analysis, topology and algebra.	[SK2] presentation/project/paper/ report
[MATMU2_W01] knows and understands in-depth the theory of selected areas of mathematics	In the field of analysis on manifold, the student knows and understands in-depth the relevant theory. Especially in this field he/she can communicate about basic definitions and formulae, knows examples and counterexamples of defined objects, and correctly formulates and proves basic theorems.	[SW4] test/exam - oral or written
[MATMU2_K06] is ready to formulate opinions on basic mathematical issues	The student is ready to formulate opinions on basic mathematical issues in the field of analysis on manifolds.	[SK1] oral statement/conversation/ discussion

	Course outcome	Subject outcome	Method of verification
	[MATMU2_U04] is able, at an advanced level and including modern mathematics, to apply and present, orally and in writing, methods of at least one selected branch of mathematics	The student is able to present mathematical arguments orally and in writing at an advanced level in the field of analysis on manifolds	[SU1] oral statement/conversation/ discussion [SU5] implementation of a problem task
	[MATMU2_U05] can perform proofs in a selected field and, if necessary, also use tools from other areas of mathematics	The student is able to perform proofs in the field of analysis on manifolds, using as well standard methods of analysis, of algebra and of topology.	[SU1] oral statement/conversation/ discussion [SU8] observation of student's independent or team work

Subject contents

1. Topological and differentiable manifolds, atlas and differentiable struccture.

2. Mappings between manifolds, the rank of a mapping.
3. Definition of submanifolds.
4. Properties of immersions, submersions and embeddings.
5. Tangent spaces and -bundles, the derivative of mappings.
6. Approximation theorem.
7. Transversality.
8. The orientation of manifolds.
9. Manifolds with boundary.
10. The degree of a mappings.
11. Differential forms, integrals with respect to them, and Stokes' Theorem.

Prerequisites and co-requisites

the standard lectures on Analysis, Linear Algebra and metric Topology
from the prior years of mathematical studies

Assessment methods and criteria

Subject passing criteria	Passing threshold	Percentage of the final grade
observation of the student's attitude	51.0%	0.0%
exam	51.0%	50.0%
activity during the example classes including the presentation of homework	51.0%	50.0%

Recommended reading

Basic literature	1. John M. Lee, "Introduction to Smooth Manifolds", Springer 2. John M. Lee, "Introduction to Topological Manifolds", Springer 3. Morris W. Hirsch, "Differential Topology", Springer 4. Martin Golubitsky, Victor Guillemin, "Stable Mappings and their Singularities", Springer 5. John Milnor, "Topologia z różniczkowego punktu widzenia", PWN 6. Michael Spivak, Analiza na rozmaitościach, PWN, 20
Supplementary literature	1. M. P. do Carmo, "Differential Forms and Applications", Springer-Verlag, Berlin, Heidelberg, 1994. 2. Victor Guillemin, Alan Pollack, "Differential topology", Prentice-Hall, Englewood Cliffs, New Jersey, 1974.
eResources addresses	

Example issues/ example questions/ tasks being completed

Credit for the lecture on the basis of the results of the oral exam in the range desrcribed in the required learning outcomes.

Credit for the auditorium class on the basis of the results of the presented solutions in the range desrcribed in the required learning outcomes

Work placement

Not applicable

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