Subject card

Subject name and code	Computational Mathematics of Evolutionary Equations, PG_00174529						
Field of study	Mathematics						
Date of commencement of studies	October 2025		Academic year of realisation of subject		2025/2026		
Education level	Master’s studies		Subject group		Optional subject group		
Mode of study	full-time studies		Mode of delivery		at the university		
Year of study	1		Language of instruction		English		
Semester of study	2		ECTS credits		6.0		
Learning profile	academic		Assessment form		exam		
Conducting unit	Institute of Mathematics -> Faculty of Mathematics, Physics and Informatics -> Rector						
Name and surname of lecturer (lecturers)	Subject supervisor		dr hab. Karolina Kropielnicka				
	Teachers						
Lesson types	Lesson type	Lecture	Tutorial	Laboratory	Project	Seminar	SUM
	Number of study hours	30.0	30.0	0.0	0.0	0.0	60
	E-learning hours included: 0.0						
Learning activity and number of study hours	Learning activity	Participation in didactic classes included in study plan		Participation in consultation hours	Self-study		SUM
	Number of study hours	60		10.0	80.0		150
Subject objectives	The aim of the course is to introduce students to numerical methods for solving evolutionary equations, i.e., equations that describe phenomena evolving in time and space, commonly arising in physics, biology, chemistry, and engineering. The course covers spatial and temporal discretisation techniques (e.g., finite difference, finite element, and spectral methods) and investigates the numerical algorithms used to solve such models. Special emphasis is placed on understanding key algorithmic properties such as accuracy, stability, efficiency, and robustness.						

Learning outcomes	Course outcome	Subject outcome	Method of verification
	[MATMU2_U03] can understand mathematical texts of various types from selected fields of mathematics	The student is able to comprehend mathematical texts of various types, especially related to numerical methods and evolutionary equations, enabling effective learning and independent problem solving.	[SU1] oral statement/conversation/ discussion
	[MATMU2_U05] can perform proofs in a selected field and, if necessary, also use tools from other areas of mathematics	The student is able to carry out mathematical proofs within numerical methods for evolutionary equations, applying tools from other branches of mathematics when necessary.	[SU4] test/exam - oral or written
	[MATMU2_K05] is ready to independently search for information in literature, also in foreign languages	The student is prepared to independently search for and use mathematical and computational literature, including in English, to deepen their knowledge and solve numerical problems.	[SK8] observation of student's independent or team work
	[MATMU2_W01] knows and understands in-depth the theory of selected areas of mathematics	The student has an in-depth understanding of the theory behind selected numerical methods for partial and ordinary differential equations, particularly in terms of stability, convergence, and error analysis.	[SW4] test/exam - oral or written
	[MATMU2_U04] is able, at an advanced level and including modern mathematics, to apply and present, orally and in writing, methods of at least one selected branch of mathematics	The student is able to apply advanced numerical methods from computational mathematics related to evolutionary equations and clearly present them in both written and oral forms.	[SU1] oral statement/conversation/ discussion [SU4] test/exam - oral or written
	[MATMU2_W02] knows and understands well the role and importance of the construction of mathematical reasoning	The student understands the importance and role of mathematical reasoning in formulating, analyzing, and verifying numerical methods applied to evolutionary equations.	[SW1] oral statement/ conversation/discussion
	[MATMU2_K01] is willing to acknowledge the limitations of his or her own knowledge and is willing to pursue further education	The student is aware of the complexity of numerical methods for evolutionary equations and the limitations of their own knowledge in this field, and demonstrates readiness for continued self-directed learning and development of computational and mathematical skills.	[SK4] test/exam - oral or written
	[MATMU2_U07] is able to define his/her interests and develop them; in particular, is able to establish contact with specialists in his/her field, e.g. understand their lectures intended for young mathematicians	The student is able to identify and develop interests in computational mathematics and evolutionary equations, including establishing contact and understanding lectures given by specialists aimed at young researchers.	[SU1] oral statement/conversation/ discussion
	[MATMU2_K06] is ready to formulate opinions on basic mathematical issues	The student is prepared to formulate informed and justified opinions on fundamental mathematical issues related to numerical methods and evolutionary equations.	[SK8] observation of student's independent or team work
	[MATMU2_U06] is able to apply methods and examples from a selected field of mathematics in related fields	The student is able to apply numerical methods and examples from computational mathematics of evolutionary equations in related fields of mathematics and applied sciences.	[SU4] test/exam - oral or written
	[MATMU2_U01] can construct mathematical reasoning: prove theorems and refute hypotheses through construction and selection of counterexamples	The student is able to formulate and justify mathematical statements concerning the properties of numerical methods (such as stability, convergence, or energy preservation), and to verify them using examples and counterexamples.	[SU4] test/exam - oral or written

	Course outcome	Subject outcome	Method of verification
	[MATMU2_W03] knows and understands in-depth a selected field of theoretical or applied mathematics and is able to understand the formulations of issues in this field that are still at the research stage and knows the connections of issues in this field with other areas of mathematics	The student has an in-depth understanding of computational methods applied to evolutionary equations, recognizes their connections with other branches of mathematics (such as functional analysis, linear algebra, or complex analysis), and is able to interpret and analyze research-level problems in this field.	[SW4] test/exam - oral or written
	[MATMU2_K04] is ready to understand and appreciate the importance of intellectual honesty in one's own and other people's actions; ethical conduct	The student understands the importance of intellectual honesty in computational and research work, is able to properly attribute sources of used methods, and acts ethically in coding, result analysis, and collaboration with others.	[SK1] oral statement/conversation/ discussion
	[MATMU2_K02] is ready to precisely formulate questions to deepen his/her understanding of a given topic or find missing elements of reasoning	The student is able to formulate precise questions related to numerical methods and models of evolutionary equations in order to deepen their understanding of the topic and to identify gaps in reasoning or computation.	[SK4] test/exam - oral or written

Subject contents	
Subject contents	1. **Introduction to Evolutionary Equations:** • Examples from wave, diffusion, and Schrödinger-type problems. 2. **Foundations of Computational Mathematics for Evolution Equations:** • Space-time discretisation: grid concepts, time/space steps. • Algorithmic quality criteria: accuracy, stability, convergence, efficiency. 3. **Time Discretisation:** • Explicit and implicit methods (Euler, Crank-Nicolson, Runge-Kutta). • Stability and stiffness (A-stability, L-stability). 4. **Space Discretisation:** • Finite difference methods (FDM). • Finite element methods (FEM). • Introduction to spectral methods and applications. 5. **Solving Evolutionary Equations:** • Coupling space and time discretisation. • Schemes for parabolic, hyperbolic, and Schrödinger-type equations. 6. **Numerical Analysis of Algorithms:** • Error sources: truncation, rounding. • CFL condition and its implications. • Conservation of physical quantities (e.g. energy) in numerical schemes. 7. **Implementation and Visualisation:** • Python implementation (NumPy, Matplotlib, SciPy). • Visualising solutions time evolution, spatial profiles.

	8. **Selected Topics and Student Projects:** • Applications to realistic problems (e.g. wave propagation, heat transfer). • Projects using real-world physical or biological models.
Prerequisites and co-requisites	**Prerequisites:** • Basic knowledge of calculus (derivatives, integrals, elementary differential equations). • Familiarity with linear algebra (systems of linear equations, matrices, vector spaces). • Basic programming skills in Python (variables, loops, functions, use of numerical libraries like NumPy). • Ability to communicate in English at a level sufficient for following lectures and participating in exercises. **Additional recommended preparation:** • Completion of the *Foundations of Computational Mathematics* course (not required but helpful). • Familiarity with basic numerical methods (e.g. Eulers method, interpolation, numerical integration). • Interest in applications of mathematics to natural sciences and engineering.

Assessment methods and criteria

Subject passing criteria	Passing threshold	Percentage of the final grade
observation of the student's attitude	51.0%	0.0%
exam	51.0%	100.0%

Recommended reading

Basic literature	1. Ben Leimkuhler, Sebastian Reich, Simulating Hamiltonian dynamics, CUP, 2004. 2. C. Lubich, From Quantum to Classical Molecular Dynamics: Reduced Models and Numerical Analysis, EMS, 2008.
Supplementary literature	1. Ben Leimkuhler, Sebastian Reich, Simulating Hamiltonian dynamics, CUP, 2004. 2. C. Lubich, From Quantum to Classical Molecular Dynamics: Reduced Models and Numerical Analysis, EMS, 2008.
eResources addresses	

Example issues/ example questions/ tasks being completed	• Differences between explicit and implicit time discretisation methods. • Numerical stability the concept of A-stability and its importance. • Space discretisation: comparison of finite difference, finite element, and spectral methods.
Work placement	Not applicable