University of Gdańsk

Subject card

Subject name and code	Theory of knots, PG_00179031						
Field of study	Mathematics						
Date of commencement of studies	October 2024		Academic year of realisation of subject	2025/2026			
Education level	Master's studies		Subject group	Optional subject group			
Mode of study	full-time studies		Mode of delivery	at the university			
Year of study	2		Language of instruction	Polish			
Semester of study	4		ECTS credits	7.0			
Learning profile	academic		Assessment form	credit			
Conducting unit							
Name and surname of lecturer (lecturers)	Subject supervisor		dr hab. Maciej Mroczkowski				
	Teachers						
Lesson types	Lesson type	Lecture	Tutorial	Laboratory	Project	Seminar	SUM
	Number of study hours	0.0	0.0	0.0	0.0	30.0	30
	E-learning hours included: 0.0						
Learning activity and number of study hours	Learning activity	Participation in didactic classes included in study plan		Participation in consultation hours	Self-study	SUM	
	Number of study hours	30		10.0	135.0	175	
Subject objectives	We will study mathematical knots that model knots that we encounter in our environment. We will see how knots can be formally written using diagrams, and in turn, knot diagrams can be encoded in various ways. One of the basic concepts will be the so-called Reidemeister moves, which allow us to move between different diagrams of the same knot. With the help of these moves, we will define various knot invariants, i.e. functions that assign to each knot an element of some algebraic object, e.g. a set of integers or polynomials. Such functions are often defined on knot diagrams, but we require that they have the same value on different diagrams of the same knot. Important examples of such invariants that we will consider are: the Jones polynomial, the Conway-Alexander polynomial, and the Fox n-coloring. Another topic will be the fundamental group of a knot. To introduce this concept, we will need to learn some elements of group theory and algebraic topology.						

Learning outcomes

Course outcome	Subject outcome	Method of verification
[MATMU2_U05] can perform proofs in a selected field and, if necessary, also use tools from other areas of mathematics	The student can apply methods from algebra, especially group theory, in proofs of theorems concerning knot theory.	[SU2] presentation/project/paper/ report [SU3] text preparation/written work
[MATMU2_K01] is willing to acknowledge the limitations of his or her own knowledge and is willing to pursue further education	The student knows which aspects of knot theory require a deeper understanding of algebraic topology and is motivated to deepen his or her knowledge in this area.	[SK1] oral statement/conversation/ discussion
[MATMU2_U04] is able, at an advanced level and including modern mathematics, to apply and present, orally and in writing, methods of at least one selected branch of mathematics	The student is able to present contemporary issues in knot theory orally and in writing.	[SU2] presentation/project/paper/ report [SU3] text preparation/written work
[MATMU2_W07] knows and understands the basic legal and ethical conditions related to scientific and teaching activities	The student understands ethical aspects in writing a master's thesis, e.g. avoiding plagiarism.	[SW3] text preparation/written work
[MATMU2_W03] knows and understands in-depth a selected field of theoretical or applied mathematics and is able to understand the formulations of issues in this field that are still at the research stage and knows the connections of issues in this field with other areas of mathematics	The student knows the basic concepts of knot theory and group theory; she/he understands the connections between knot theory and algebra and is able to understand the topics of some research works in the field of knot theory.	[SW4] test/exam - oral or written [SW2] presentation/project/paper/ report [SW3] text preparation/written work
[MATMU2_K06] is ready to formulate opinions on basic mathematical issues	The student formulates opinions on his or her favorite or least favorite aspects of knot theory and group theory.	[SK1] oral statement/conversation/ discussion
[MATMU2_U02] can express mathematical content in speech and writing, in mathematical texts of various nature	The student presents papers and writes a master's thesis on knot theory in a clear manner.	[SU2] presentation/project/paper/ report [SU3] text preparation/written work
[MATMU2_U03] can understand mathematical texts of various types from selected fields of mathematics	The student understands selected chapters from books on knot theory and selected research papers.	[SU1] oral statement/conversation/ discussion [SU2] presentation/project/paper/ report
[MATMU2_U07] is able to define his/her interests and develop them; in particular, is able to establish contact with specialists in his/her field, e.g. understand their lectures intended for young mathematicians	The student is interested in occasional lectures at seminars or conferences related to knot theory or group theory.	[SU1] oral statement/conversation/ discussion
[MATMU2_U08] is able to prepare in-depth written works in Polish and at least one foreign language in a selected field of mathematics	The student writes a master's thesis in Polish and is able to prepare papers in English, using the literature on knot theory in that language.	[SU1] oral statement/conversation/ discussion [SU3] text preparation/written work
[MATMU2_K02] is ready to precisely formulate questions to deepen his/her understanding of a given topic or find missing elements of reasoning	The student is able to ask questions regarding his or her doubts in understanding the issues and proofs related to knot theory.	[SK1] oral statement/conversation/ discussion
[MATMU2_K04] is ready to understand and appreciate the importance of intellectual honesty in one's own and other people's actions; ethical conduct	The student appreciates honesty in writing work and preparing reports without plagiarism; is able to admit that she or he does not understand some aspects in the proves of theorems and does not pretend to understand these aspects.	[SK1] oral statement/conversation/ discussion
[MATMU2_K05] is ready to independently search for information in literature, also in foreign languages	The student searches by herfself or himself for literature on knot theory in Polish and English.	[SK1] oral statement/conversation/ discussion
[MATMU2_U09] can prepare oral presentations in-depth in Polish and at least one foreign language, in a selected field of mathematics	The student presents talks on knot theory in Polish or English.	[SU2] presentation/project/paper/ report

<table>
<tr><td>Subject contents</td><td>- definition of knots and links
- knot and tangle diagrams with different ways of encoding them
- Reidemeister moves on diagrams
- Fox n-colorings on knots
- Conway-Alexander polynomial
- Jones polynomial
- other important knot invariants
- fundamental group, group presentation
- knot group, Wirtinger presentation
- studying groups with given presentations</td></tr>
<tr><td>Prerequisites and co-requisites</td><td>Elementary knowledge of analysis, linear algebra. It is good if the student knows the basics of abstract algebra, although this will also be developed during the seminar.</td></tr>
<tr><td>Assessment methods and criteria</td><td><table>
<tr><th>Subject passing criteria</th><th>Passing threshold</th><th>Percentage of the final grade</th></tr>
<tr><td>master's thesis</td><td>51.0%</td><td>50.0%</td></tr>
<tr><td>student's talk</td><td>51.0%</td><td>50.0%</td></tr>
</table></td></tr>
<tr><td>Recommended reading</td><td><table>
<tr><td>Basic literature</td><td>C. Adams, The Knot Book. An Elementary Introduction to the Mathematical Theory of Knots, AMS 2004.
C. Livingston, Knot Theory, MAA 1993.
W.B.R Lickorish, An Introduction to Knot Theory, Springer 1997.
J.H. Przytycki, Teoria wezłów i zwiazanych z nimi struktur dystrybutywnych, Wydawnictwo UG 2016.
D. Rolfsen, Knots and links, AMS Chealsea Publishing</td></tr>
<tr><td>Supplementary literature</td><td>G. Burde , H. Zieschang and M. Heusener, Knots, De Gruyter, 2013

K. Murasugi, Knot Theory and Its Application, Springer 1993</td></tr>
<tr><td>eResources addresses</td><td></td></tr>
</table></td></tr>
<tr><td>Example issues/ example questions/ tasks being completed</td><td>- Calculating knot polynomials and distinguishing knots
- Calculating knot groups and analyzing these groups</td></tr>
<tr><td>Work placement</td><td>Not applicable</td></tr>
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