

Subject card

Subject name and code	Clifford algebras, PG_00179024						
Field of study	Mathematics						
Date of commencement of studies	October 2025		Academic year of realisation of subject		2025/2026		
Education level	Master's studies		Subject group				
Mode of study	full-time studies		Mode of delivery		at the university		
Year of study	1		Language of instruction		Polish polish or english		
Semester of study	1		ECTS credits		5.0		
Learning profile	academic		Assessment form		credit		
Conducting unit							
Name and surname of lecturer (lecturers)	Subject supervisor		dr hab. Ewa Tyszkowska				
	Teachers						
Lesson types	Lesson type	Lecture	Tutorial	Laboratory	Project	Seminar	SUM
	Number of study hours	0.0	0.0	0.0	0.0	30.0	30
	E-learning hours included: 0.0						
Learning activity and number of study hours	Learning activity	Participation in didactic classes included in study plan		Participation in consultation hours		Self-study	SUM
	Number of study hours	30		5.0		90.0	125
Subject objectives	The aim of the course is to familiarize students with the topic of Clifford algebras and to explain how rotations in R^n are induced by the action of a certain group, $Spin(n)$, on R^n , in a way that generalizes the actions of complex numbers with modulus 1 on R^2 and the actions of unit quaternions on R^3 . The group $Spin(n)$, called the spin group, is defined as the group of units of the Clifford algebra Cl_n associated with R^n . The seminar is dedicated to Clifford algebras and the groups $Spin$ and Pin , including the structures of the Clifford algebra $Cl(p,q)$ associated with a non-degenerate symmetric bilinear form with signature (p,q) as well as the theorem on the 8-periodicity of such algebras.						

Learning outcomes	Course outcome	Subject outcome	Method of verification
	[MATMU2_W03] knows and understands in-depth a selected field of theoretical or applied mathematics and is able to understand the formulations of issues in this field that are still at the research stage and knows the connections of issues in this field with other areas of mathematics	The student has an in-depth knowledge and understanding of Clifford algebras and their connections with other areas of mathematics.	[SW2] presentation/project/paper/report
	[MATMU2_U09] can prepare oral presentations in-depth in Polish and at least one foreign language, in a selected field of mathematics	The student is able to prepare and deliver a presentation in Polish and in English on a given topic from the studied field.	[SU2] presentation/project/paper/report
	[MATMU2_W07] knows and understands the basic legal and ethical conditions related to scientific and teaching activities	The student knows and understands the basic legal and ethical conditions related to scientific and educational activities	[SW2] presentation/project/paper/report
	[MATMU2_U08] is able to prepare in-depth written works in Polish and at least one foreign language in a selected field of mathematics	The student is able to prepare in-depth written works in Polish and at least one foreign language, in a field of Clifford algebras	[SU3] text preparation/written work
	[MATMU2_U02] can express mathematical content in speech and writing, in mathematical texts of various nature	The student is able to discuss mathematical content from a given topic both orally and in writing	[SU2] presentation/project/paper/report [SU3] text preparation/written work
	[MATMU2_K06] is ready to formulate opinions on basic mathematical issues	The student is able to express opinions on basic mathematical topics.	[SK1] oral statement/conversation/discussion
	[MATMU2_U04] is able, at an advanced level and including modern mathematics, to apply and present, orally and in writing, methods of at least one selected branch of mathematics	The student is able to present orally and in writing the methods used in studying Clifford algebra.	[SU1] oral statement/conversation/discussion [SU2] presentation/project/paper/report
	[MATMU2_K04] is ready to understand and appreciate the importance of intellectual honesty in one's own and other people's actions; ethical conduct	The student understands and appreciates the importance of intellectual honesty in their own actions and those of others, and behaves ethically	[SK8] observation of student's independent or team work
	[MATMU2_U03] can understand mathematical texts of various types from selected fields of mathematics	The student is able to understand mathematical texts concerning Clifford algebras	[SU1] oral statement/conversation/discussion
	[MATMU2_U05] can perform proofs in a selected field and, if necessary, also use tools from other areas of mathematics	The student is able to prove the discussed theorems, using tools from other branches of mathematics if necessary	[SU1] oral statement/conversation/discussion
	[MATMU2_K05] is ready to independently search for information in literature, also in foreign languages	The student is able to find the necessary information in literature both in their native language and in foreign languages.	[SK2] presentation/project/paper/report [SK3] text preparation/written work
	[MATMU2_K02] is ready to precisely formulate questions to deepen his/her understanding of a given topic or find missing elements of reasoning	The student is able to precisely formulate questions aimed at deepening their understanding of a given topic or identifying missing elements in their reasoning	[SK1] oral statement/conversation/discussion
	[MATMU2_K01] is willing to acknowledge the limitations of his or her own knowledge and is willing to pursue further education	The student is aware of the limitations of their own knowledge and understands the need for further education	[SK8] observation of student's independent or team work
	[MATMU2_U07] is able to define his/her interests and develop them; in particular, is able to establish contact with specialists in his/her field, e.g. understand their lectures intended for young mathematicians	The student is able to identify their interests and develop them through reading literature or listening of lectures related to those interests	[SU8] observation of student's independent or team work

Subject contents	• Tensor algebras. Exterior algebras. • Quadratic forms. • Orthogonal groups $O(Q)$ and $SO(Q)$. • Maximal orthogonal subspace. Witt decomposition. • Definitions of Clifford algebra $Cl(Q)$. Examples of low-dimensional real Clifford algebras. • The universal property of Clifford algebra • Basis of the Clifford algebra. • Center of $Cl(Q)$ and $Cl(Q)^+$. • Classification of real and complex Clifford algebras. • Clifford groups. • Spin and Pin groups. • Spinor representations of $Cl(Q)$. Dirac matrices. • Spinor representations of $Cl(Q)^+$. Pauli matrices. • Periodicity of Clifford algebras $Cl(p,q)$. • $Pin(p,q)$ and $Spin(p,q)$ groups as double covers of orthogonal groups. • Spin structures on manifolds (if time permits).		
Prerequisites and co-requisites	Basic concepts of linear algebra and general algebra		
Assessment methods and criteria	Subject passing criteria	Passing threshold	Percentage of the final grade
	Observation of the student's attitude	51.0%	0.0%
	Written works	51.0%	45.0%
	Reporting on assigned topics	51.0%	45.0%
	Student activity during classes	51.0%	10.0%
Recommended reading	Basic literature	1) Clifford algebras, Clifford groups, and generalization of the Quaternions: The Pin and Spin Groups, Jean Galier https://doi.org/10.48550/arXiv.0805.0311 2) Spin Geometry, Jose Figueroa-O'Farril, http://empg.maths.ed.ac.uk/Activities/Spin 3) Od liczb zespolonych do tensorów, spinorów, algebr Liego i kwadryk, Jacek Komorowski.	
	Supplementary literature	none	
	eResources addresses	Basic https://doi.org/10.48550/arXiv.0805.0311 - Clifford algebras, Clifford groups, and generalization of the Quaternions: The Pin and Spin Groups, Jean Galier http://empg.maths.ed.ac.uk/Activities/Spin - Spin Geometry, Jose Figueroa-O'Farril	
Example issues/ example questions/ tasks being completed	Classification of real and complex Clifford algebras Representations of Clifford algebras		
Work placement	Not applicable		