University of Gdańsk

Subject card

Subject name and code	Clifford algebras, PG_00179213						
Field of study	Mathematics						
Date of commencement of studies	October 2025		Academic year of realisation of subject	2025/2026			
Education level	Master's studies		Subject group				
Mode of study	full-time studies		Mode of delivery	at the university			
Year of study	1		Language of instruction	Polish			
Semester of study	2		ECTS credits	5.0			
Learning profile	academic		Assessment form	credit			
Conducting unit							
Name and surname of lecturer (lecturers)	Subject supervisor		dr hab. Ewa Tyszkowska				
	Teachers						
Lesson types	Lesson type	Lecture	Tutorial	Laboratory	Project	Seminar	SUM
	Number of study hours	0.0	0.0	0.0	0.0	30.0	30
	E-learning hours included: 0.0						
Learning activity and number of study hours	Learning activity	Participation in didactic classes included in study plan		Participation in consultation hours	Self-study	SUM	
	Number of study hours	30		5.0	90.0	125	
Subject objectives	The aim of the course is to familiarize students with the topic of Clifford algebras and to explain how rotations in R^n are induced by the action of a certain group, Spin(n), on R^n, in a way that generalizes the actions of complex numbers with modulus 1 on R^2 and the actions of unit quaternions on R^3. The group Spin(n), called the spin group, is defined as the group of units of the Clifford algebra Cln associated with R^n. The seminar is dedicated to Clifford algebras and the groups Spin and Pin, including the structures of the Clifford algebra Cl(p,q) associated with a non-degenerate symmetric bilinear form with signature (p,q), as well as the theorem on the 8-periodicity of such algebras. Learning outcomes:							

Learning outcomes

Course outcome	Subject outcome	Method of verification
[MATMU2_K05] is ready to independently search for information in literature, also in foreign languages	The student is able to find the necessary information in literature both in their native language and in foreign languages.	[SK2] presentation/project/paper/ report
[MATMU2_U04] is able, at an advanced level and including modern mathematics, to apply and present, orally and in writing, methods of at least one selected branch of mathematics	The student is able to present orally and in writing the methods used in studying Clifford algebra.	[SU1] oral statement/conversation/ discussion
[MATMU2_U03] can understand mathematical texts of various types from selected fields of mathematics	The student is able to understand mathematical texts concerning Clifford algebras	[SU1] oral statement/conversation/ discussion
[MATMU2_U09] can prepare oral presentations in-depth in Polish and at least one foreign language, in a selected field of mathematics	The student is able to prepare and deliver a presentation in Polish and in English on a given topic from the studied field.	[SU2] presentation/project/paper/ report
[MATMU2_U07] is able to define his/her interests and develop them; in particular, is able to establish contact with specialists in his/her field, e.g. understand their lectures intended for young mathematicians	The student is able to identify their interests and develop them through reading literature or llistening of lectures related to those interests	[SU2] presentation/project/paper/ report
[MATMU2_W07] knows and understands the basic legal and ethical conditions related to scientific and teaching activities	The student knows and understands the basic legal and ethical conditions related to scientific and educational activities	[SW1] oral statement/ conversation/discussion
[MATMU2_W03] knows and understands in-depth a selected field of theoretical or applied mathematics and is able to understand the formulations of issues in this field that are still at the research stage and knows the connections of issues in this field with other areas of mathematics	The student has an in-depth knowledge and understanding of Clifford algebras and their connections with other areas of mathematics.	[SW2] presentation/project/paper/ report
[MATMU2_U05] can perform proofs in a selected field and, if necessary, also use tools from other areas of mathematics	The student is able to prove the discussed theorems, using tools from other branches of mathematics if necessary	[SU1] oral statement/conversation/ discussion
[MATMU2_K02] is ready to precisely formulate questions to deepen his/her understanding of a given topic or find missing elements of reasoning	The student is able to precisely formulate questions aimed at deepening their understanding of a given topic or identifying missing elements in their reasoning	[SK8] observation of student's independent or team work
[MATMU2_K06] is ready to formulate opinions on basic mathematical issues	The student is able to express opinions on basic mathematical topics.	[SK1] oral statement/conversation/ discussion
[MATMU2_U02] can express mathematical content in speech and writing, in mathematical texts of various nature	The student is able to discuss mathematical content from a given topic both orally and in writing	[SU1] oral statement/conversation/ discussion [SU3] text preparation/written work
[MATMU2_K01] is willing to acknowledge the limitations of his or her own knowledge and is willing to pursue further education	The student is aware of the limitations of their own knowledge and understands the need for further education	[SK8] observation of student's independent or team work
[MATMU2_K04] is ready to understand and appreciate the importance of intellectual honesty in one's own and other people's actions; ethical conduct	The student understands and appreciates the importance of intellectual honesty in their own actions and those of others, and behaves ethically	[SK8] observation of student's independent or team work
[MATMU2_U08] is able to prepare in-depth written works in Polish and at least one foreign language in a selected field of mathematics	The student is able to prepare in-depth written works in Polish and at least one foreign language, in a field of Clifford algebras	[SU3] text preparation/written work

Subject contents

- Tensor algebras. Exterior algebras.
- Quadratic forms.
- Orthogonal groups O(Q) and SO(Q).
- Maximal orthogonal subspace. Witt decomposition.
- Definitions of Clifford algebra Cl(Q). Examples of low-dimensional real Clifford algebras.
- The universal property of Clifford algebra
- Basis of the Clifford algebra.

- Center of Cl(Q) and Cl(Q)^+.
- Classification of real and complex Clifford algebras.
- Clifford groups.
- Spin and Pin groups.
- Spinor representations of Cl(Q). Dirac matrices.
- Spinor representations of Cl(Q)^+. Pauli matrices.
- Periodicity of Clifford algebras Cl(p,q).
- Pin(p,q) and Spin(p,q) groups as double covers of orthogonal groups.
- Spin structures on manifolds (if time permits).

Prerequisites and co-requisites

Basic concepts of linear algebra and general algebra

Assessment methods and criteria

Subject passing criteria	Passing threshold	Percentage of the final grade
Observation of student's attitude	51.0%	0.0%
Student activity during classes	51.0%	10.0%
Written works	51.0%	45.0%
Reporting on assigned topics	51.0%	45.0%

Recommended reading

Basic literature	1)Clifford algebras, Clifford groups, and generalization of the Quaternions: The Pin and Spin Groups, Jean Galier https://doi.org/10.48550/arXiv.0805.0311 2) Spin Geometry, Jose Figoreoa-O'Farril, http://empg.maths.ed.ac.uk/Activities/Spin 3) Od liczb zespolonych do tensorów, spinorów, algebr Liego i kwadryk, Jacek Komorowski.
Supplementary literature	None
eResources addresses	

Example issues/ example questions/ tasks being completed

Classification of real Clifford algebras

Representations of Clifford algebras

Work placement	Not applicable

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