

## Subject card

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| Subject name and code                       | Theory of knots, PG_00179292                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                          |                                         |                                     |         |                                                                          |     |
| Field of study                              | Mathematical Modeling and Data Analysis                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                          |                                         |                                     |         |                                                                          |     |
| Date of commencement of studies             | October 2025                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                          | Academic year of realisation of subject |                                     |         | 2025/2026                                                                |     |
| Education level                             | Master's studies                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                          | Subject group                           |                                     |         | Obligatory subject group in the field of study<br>Optional subject group |     |
| Mode of study                               | full-time studies                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                          | Mode of delivery                        |                                     |         | at the university                                                        |     |
| Year of study                               | 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                          | Language of instruction                 |                                     |         | Polish                                                                   |     |
| Semester of study                           | 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                          | ECTS credits                            |                                     |         | 5.0                                                                      |     |
| Learning profile                            | academic                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                          | Assessment form                         |                                     |         | credit                                                                   |     |
| Conducting unit                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                          |                                         |                                     |         |                                                                          |     |
| Name and surname of lecturer (lecturers)    | Subject supervisor                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                          | dr hab. Maciej Mroczkowski              |                                     |         |                                                                          |     |
|                                             | Teachers                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                          |                                         |                                     |         |                                                                          |     |
| Lesson types                                | Lesson type                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | Lecture                                                  | Tutorial                                | Laboratory                          | Project | Seminar                                                                  | SUM |
|                                             | Number of study hours                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 0.0                                                      | 0.0                                     | 0.0                                 | 0.0     | 30.0                                                                     | 30  |
|                                             | E-learning hours included: 0.0                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                          |                                         |                                     |         |                                                                          |     |
| Learning activity and number of study hours | Learning activity                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | Participation in didactic classes included in study plan |                                         | Participation in consultation hours |         | Self-study                                                               | SUM |
|                                             | Number of study hours                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 30                                                       |                                         | 5.0                                 |         | 90.0                                                                     | 125 |
| Subject objectives                          | <p>We will study mathematical knots that model knots that we encounter in our environment. We will see how knots can be formally written using diagrams, and in turn, knot diagrams can be encoded in various ways. One of the basic concepts will be the so-called Reidemeister moves, which allow us to move between different diagrams of the same knot. With the help of these moves, we will define various knot invariants, i.e. functions that assign to each knot an element of some algebraic object, e.g. a set of integers or polynomials. Such functions are often defined on knot diagrams, but we require that they have the same value on different diagrams of the same knot. Important examples of such invariants that we will consider are: the Jones polynomial, the Conway-Alexander polynomial, and the Fox n-coloring.</p> <p>Another topic will be the fundamental group of a knot. To introduce this concept, we will need to learn some elements of group theory and algebraic topology.</p> |                                                          |                                         |                                     |         |                                                                          |     |

| Learning outcomes | Course outcome                                                                                                                                                                                                                                                                                 | Subject outcome                                                                                                                                                                                                                      | Method of verification                                                                  |
|-------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------|
|                   | [MMiADMU2_K04] is ready to understand and appreciate the importance of intellectual honesty in the actions of himself and others; is ready to act ethically                                                                                                                                    | The student appreciates honesty in writing work and preparing reports without plagiarism; is able to admit to misunderstanding aspects of the proof and does not pretend to understand these aspects.                                | [SK1] oral statement/conversation/discussion                                            |
|                   | [MMiADMU2_U04] can, at an advanced level and including modern mathematics, apply and present in speech and in writing the methods of at least one selected branch of mathematics                                                                                                               | The student is able to present contemporary issues in knot theory orally and in writing.                                                                                                                                             | [SU2] presentation/project/paper/report<br>[SU3] text preparation/written work          |
|                   | [MMiADMU2_K06] is ready to form an opinion on basic mathematical issues                                                                                                                                                                                                                        | The student formulates opinions on his or her favorite or least favorite aspects of knot theory and group theory.                                                                                                                    | [SK1] oral statement/conversation/discussion                                            |
|                   | [MMiADMU2_K02] is ready to precisely formulate questions to deepen his own understanding of a given topic or to find missing elements of reasoning                                                                                                                                             | The student is able to ask questions regarding his or her doubts in understanding the issues and proofs of the theorems related to knot theory.                                                                                      | [SK1] oral statement/conversation/discussion                                            |
|                   | [MMiADMU2_U03] can understand mathematical texts from selected areas of mathematics                                                                                                                                                                                                            | The student understands selected chapters from books on knot theory and selected research papers.                                                                                                                                    | [SU1] oral statement/conversation/discussion<br>[SU2] presentation/project/paper/report |
|                   | [MMiADMU2_U07] is able to define his interests and develop them; in particular, is able to establish contact with specialists in their field, e.g. understand their lectures intended for young mathematicians                                                                                 | The student is interested in occasional lectures at seminars or conferences related to knot theory or group theory.                                                                                                                  | [SU1] oral statement/conversation/discussion                                            |
|                   | [MMiADMU2_U09] can prepare in-depth oral presentations in Polish and at least in one foreign language in a selected field of mathematics                                                                                                                                                       | The student presents papers on knot theory in Polish or English.                                                                                                                                                                     | [SU2] presentation/project/paper/report                                                 |
|                   | [MMiADMU2_U08] is able to in-depth preparation of texts in Polish and at least one foreign language, in a selected field of mathematics                                                                                                                                                        | The student writes a master's thesis in Polish and is able to prepare papers in English, using the literature on knot theory in that language.                                                                                       | [SU1] oral statement/conversation/discussion<br>[SU3] text preparation/written work     |
|                   | [MMiADMU2_W07] knows and understands the basic legal and ethical conditions related to research and teaching                                                                                                                                                                                   | The student understands ethical aspects in writing a master's thesis, e.g. avoiding plagiarism.                                                                                                                                      | [SW3] text preparation/written work                                                     |
|                   | [MMiADMU2_U02] can express mathematical content in speech and in writing, in mathematical texts of various nature                                                                                                                                                                              | The student presents papers and writes a master's thesis on knot theory in a clear manner.                                                                                                                                           | [SU2] presentation/project/paper/report<br>[SU3] text preparation/written work          |
|                   | [MMiADMU2_W03] knows and understands in-depth a selected field of theoretical or applied mathematics and is able to understand the formulations of issues in this field that are still being researched and knows the links between the issues of this field and other branches of mathematics | The student knows the basic concepts of knot theory and group theory; she or he understands the connections between knot theory and algebra and is able to understand the topics of some research works in the field of knot theory. | [SW2] presentation/project/paper/report<br>[SW3] text preparation/written work          |
|                   | [MMiADMU2_U05] is able to make proofs and analyzes in a given field, in which, if necessary, he also uses tools from other areas of mathematics                                                                                                                                                | The student can apply methods from algebra, especially group theory, in proofs of theorems concerning knot theory.                                                                                                                   | [SU2] presentation/project/paper/report<br>[SU3] text preparation/written work          |
|                   | [MMiADMU2_K01] is ready to recognize the limitations of his own knowledge and is ready to continue his education                                                                                                                                                                               | The student knows which aspects of knot theory require a deeper understanding of algebraic topology and is motivated to deepen his or her knowledge in this area.                                                                    | [SK1] oral statement/conversation/discussion                                            |
|                   | [MMiADMU2_K05] is ready to independently search for information in literature, also in foreign languages                                                                                                                                                                                       | The student searches by herself/himself for literature on knot theory in Polish and English.                                                                                                                                         | [SK1] oral statement/conversation/discussion                                            |

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| Subject contents                                               | <ul style="list-style-type: none"><li>- definition of knots and links</li><li>- knot and tangle diagrams with different ways of encoding them</li><li>- Reidemeister moves on diagrams</li><li>- Fox n-colorings on knots</li><li>- Conway-Alexander polynomial</li><li>- Jones polynomial</li><li>- other important knot invariants</li><li>- fundamental group, group presentation</li><li>- knot group, Wirtinger presentation</li><li>- studying groups with given presentations</li></ul> |                                                                                                                                                                                                                                                                                                                                                                            |                               |
| Prerequisites and co-requisites                                | Elementary knowledge of analysis, linear algebra. It is good if the student knows the basics of abstract algebra, although this will also be developed during the seminar.                                                                                                                                                                                                                                                                                                                     |                                                                                                                                                                                                                                                                                                                                                                            |                               |
| Assessment methods and criteria                                | Subject passing criteria                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Passing threshold                                                                                                                                                                                                                                                                                                                                                          | Percentage of the final grade |
|                                                                | student's talks with submitted written versions                                                                                                                                                                                                                                                                                                                                                                                                                                                | 51.0%                                                                                                                                                                                                                                                                                                                                                                      | 80.0%                         |
|                                                                | short written tests                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | 51.0%                                                                                                                                                                                                                                                                                                                                                                      | 20.0%                         |
|                                                                | observation of student's attitude                                                                                                                                                                                                                                                                                                                                                                                                                                                              | 51.0%                                                                                                                                                                                                                                                                                                                                                                      | 0.0%                          |
| Recommended reading                                            | Basic literature                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | C. Adams, The Knot Book. An Elementary Introduction to the Mathematical Theory of Knots, AMS 2004.<br>C. Livingston, Knot Theory, MAA 1993.<br>W.B.R Lickorish, An Introduction to Knot Theory, Springer 1997.<br>J.H. Przytycki, Teoria węzłów i związanych z nimi struktur dystrybutywnych, Wydawnictwo UG 2016.<br>D. Rolfsen, Knots and links, AMS Chealsea Publishing |                               |
|                                                                | Supplementary literature                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | G. Burde , H. Zieschang and M. Heusener, Knots, De Gruyter, 2013<br><br>K. Murasugi, Knot Theory and Its Application, Springer 1993                                                                                                                                                                                                                                        |                               |
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|                                                                | eResources addresses                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                            |                               |
| Example issues/<br>example questions/<br>tasks being completed | <ul style="list-style-type: none"><li>- Calculating knot polynomials and distinguishing knots</li><li>- Calculating knot groups and analyzing these groups</li></ul>                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                            |                               |
| Work placement                                                 | Not applicable                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                                                                                                                                                                                                                                                                                                                                                                            |                               |

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