University of Gdańsk

Subject card

Subject name and code	Knots and their invariants, PG_00179451						
Field of study	Mathematical Modeling and Data Analysis						
Date of commencement of studies	October 2023		Academic year of realisation of subject	2025/2026			
Education level	Bachelor's studies		Subject group	Obligatory subject group in the field of study Optional subject group			
Mode of study	full-time studies		Mode of delivery	at the university			
Year of study	3		Language of instruction	Polish			
Semester of study	6		ECTS credits	6.0			
Learning profile	academic		Assessment form	credit			
Conducting unit							
Name and surname of lecturer (lecturers)	Subject supervisor		dr hab. Maciej Mroczkowski				
	Teachers						
Lesson types	Lesson type	Lecture	Tutorial	Laboratory	Project	Seminar	SUM
	Number of study hours	0.0	0.0	0.0	0.0	30.0	30
	E-learning hours included: 0.0						
Learning activity and number of study hours	Learning activity	Participation in didactic classes included in study plan		Participation in consultation hours		Self-study	SUM
	Number of study hours	30		0.0		0.0	30
Subject objectives	We will study mathematical knots that model the knots we encounter in our environment. We will see how knots can be formally written using diagrams, and in turn, knot diagrams can be encoded in different ways. One of the basic concepts will be the so-called Reidemeister moves, which allow us to move between different diagrams of the same knot. With the help of these moves, we will define various knot invariants, i.e. functions that assign each knot an element of some algebraic object, e.g. a set of integers or polynomials. Important examples of such invariants that we will consider are: the Jones polynomial, the Conway-Alexander polynomial, and the Fox n-coloring. After defining several invariants and examining their properties, we will use Python programs to analyze the values of such invariants on large sets of knots (available in databases on the web).						

Learning outcomes

Course outcome	Subject outcome	Method of verification
[MMiADL3_W13] knows and understands the basic concepts and principles of industrial property protection and copyright, is able to use patent information resources	The student is familiar with the issue of copyright and various licenses relating to the books and works she or he is using.	[SW1] oral statement/ conversation/discussion
[MMiADL3_K06] is ready to formulate opinions on basic mathematical issues	The student formulates opinions on his or her favorite or least favorite aspects of knot theory and group theory.	[SK1] oral statement/conversation/ discussion
[MMiADL3_K10] is ready to analyse data and communicate the conclusions of such analysis in an accessible form	The student is able to write a Python program to analyze data about knots. Then, the student is able to present the results of this analysis to the group in an accessible way.	[SK1] oral statement/conversation/ discussion [SK2] presentation/project/paper/ report
[MMiADL3_U15] can speak about mathematical problems in understandable colloquial language	During her or his talk, the student uses comparisons of concepts from knot theory with understandable everyday concepts or concepts from other, less advanced areas of mathematics.	[SU2] presentation/project/paper/ report
[MMiADL3_W12] has a basic knowledge and understanding of legal and ethical conditions related to scientific and didactic activities	The student cites as many sources as possible that he/she uses in his/her reports and is familiar with copyright issues.	[SW1] oral statement/ conversation/discussion
[MMiADL3_U14] has the ability to prepare oral presentations in Polish and at least one foreign language about selected mathematical issues using various sources of knowledge	The student prepares presentations on knot theory in Polish and is able to do so in English.	[SU2] presentation/project/paper/ report
[MMiADL3_K04] is ready to understand and appreciate the importance of intellectual honesty in own and other people's actions; is ready to act ethically	The student is able to admit to not understanding aspects of a proof and does not pretend to understand those aspects.	[SK1] oral statement/conversation/ discussion
[MMiADL3_K01] is ready to accept the limitations of his/her own knowledge and understands the need for further education	The student knows which aspects of knot theory require a deeper understanding of algebraic topology and is motivated to deepen his or her knowledge in this area.	[SK1] oral statement/conversation/ discussion
[MMiADL3_K09] is ready to critically evaluate arguments, find gaps in reasoning, and constructively criticise other people's reasoning	The student notices flawed reasoning in other students' reports or the lecturer's explanations and is able to justify his/her reservations.	[SK1] oral statement/conversation/ discussion [SK2] presentation/project/paper/ report
[MMiADL3_K02] is ready to precisely formulate questions to deepen his/her own understanding of a given topic or to find missing elements of reasoning	The student is able to ask questions regarding his or her doubts in understanding the issues and proofs of theorems from knot theory.	[SK1] oral statement/conversation/ discussion

Subject contents

- definition of knots and links
- knot and tangle diagrams with different ways of encoding them
- Reidemeister moves on diagrams
- Fox n-colorings on knots
- Conway-Alexander polynomial
- Jones polynomial - other knot invariants
- analysis of knot data from KnotInfo

Prerequisites and co-requisites

Calculus and linear algebra

Assessment methods and criteria

Subject passing criteria	Passing threshold	Percentage of the final grade
talks	51.0%	100.0%

Recommended reading

Basic literature

C. Adams, The Knot Book. An Elementary Introduction to the Mathematical Theory of Knots, AMS 2004.
C. Livingston, Knot Theory, MAA 1993.
W.B.R Lickorish, An Introduction to Knot Theory, Springer 1997.
J.H. Przytycki, Teoria wezłów i zwiazanych z nimi struktur dystrybutywnych, Wydawnictwo UG 2016.
D. Rolfsen, Knots and links, AMS Chealsea Publishing

	Supplementary literature	W.B.R Lickorish, An Introduction to Knot Theory, Springer 1997. K. Murasugi, Knot Theory and Its Applications, Springer 1993
	eResources addresses	
Example issues/ example questions/ tasks being completed	- study of bridge numbers and braid numbers of knots - Jones polynomial analysis for knots up to 13 crossings - Fox n-coloring analysis for knots up to 13 crossings	
Work placement	Not applicable	

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