University of Gdańsk

Subject card

Subject name and code	Algebra, PG_00208373		
Field of study	Physics		
Date of commencement of studies	October 2026	Academic year of realisation of subject	2026/2027
Education level	Bachelor's studies	Subject group	Obligatory subject group in the field of study
Mode of study	full-time studies	Mode of delivery	at the university
Year of study	1	Language of instruction	Polish
Semester of study	2	ECTS credits	8.0
Learning profile	academic	Assessment form	exam
Conducting unit			
Name and surname of lecturer (lecturers)	Subject supervisor	dr hab. Adam Rutkowski, prof. UG	
	Teachers		

Lesson types

Lesson type	Lecture	Tutorial	Laboratory	Project	Seminar	SUM
Number of study hours	45.0	45.0	15.0	0.0	0.0	105

E-learning hours included: 0.0

Learning activity and number of study hours

Learning activity	Participation in didactic classes included in study plan	Participation in consultation hours	Self-study	SUM
Number of study hours	105	0.0	95.0	200

Subject objectives

The aim of the course is to familiarize students with fundamental concepts of algebra that allow the introduction and analysis of linear spaces and linear mappings between them. The course covers topics related to matrices and determinants, systems of linear equations, dual spaces, eigenvalue problems, and quadratic forms. The lectures provide a clear presentation of these concepts and theorems using finite-dimensional vector spaces, whose generalizations play a key role in Hilbert space theory. Understanding these topics facilitates the later introduction of ideas relevant to physical applications.

Learning outcomes

Course outcome	Subject outcome	Method of verification
[FIZL3_W04] knows, on the advanced level, the methods of higher mathematics, including differential and integral calculus of functions of one and many variables, and the basics of algebra to the extent necessary to describe physical phenomena and solve physical problems	The student knows the concepts of linear and abstract algebra, such as matrices, determinants, vector spaces, groups, rings, and fields. They understand the properties of algebraic operations and the relationships between algebraic structuressolving physical problems.	[SW4] test/exam - oral or written [SW1] oral statement/ conversation/discussion [SW2] presentation/project/paper/ report [SW3] text preparation/written work [SW5] implementation of a problem task
[FIZL3_U01] can use advanced mathematical formalism to define, describe, and solve problems in physics	The student can perform operations on matrices, calculate determinants, eigenvalues, and eigenvectors. They can examine the properties of groups, rings, and fields and apply them to solve algebraic problems. They can use the tools of linear and abstract algebra to model physical and mathematical problems. They can prove simple algebraic theorems and justify the correctness of their solutions.	[SU1] oral statement/conversation/ discussion [SU3] text preparation/written work [SU4] test/exam - oral or written [SU5] implementation of a problem task

Subject contents

1. **Introductory topics:** logical and set-theoretic notation, functions, mathematical induction, summation and product symbols
2. **Algebraic structures:** groups: operations, properties of operations, definition of the simplest group, permutation group, subgroups; rings and fields: definition and the simplest properties of a ring, definition and the simplest properties of a field, subrings and subfields
3. **Isomorphisms and homomorphisms:** isomorphisms of sets with operations (groups and rings), homomorphisms of sets with operations (groups and rings), isomorphic embeddings
4. **The field of complex numbers:** construction of the field of complex numbers, complex conjugates and modulus, geometric interpretation of complex numbers, trigonometric and exponential form of a complex number, roots of a complex number
5. **Matrices and determinants:** operations on matrices, matrix algebra, definitions and properties of the determinant: various methods of calculating the determinant, inverse matrix
6. **Systems of linear equations:** general theory of systems of linear equations, Cramer systems, homogeneous systems, solving systems of linear equations by the Gaussian elimination method
7. **Vector spaces:** definition and basic properties of vector spaces, subspaces, linear dependence and independence of vectors, span, basis and dimension of a vector space, isomorphisms of vector spaces
8. **Homomorphisms of vector spaces:** definition of a homomorphism of vector spaces, algebra of endomorphisms of a vector space, matrix representation of endomorphisms, properties of endomorphisms, eigenvectors and eigenvalues, change of basis
9. **Functionals and forms:** linear and bilinear functionals and forms, quadratic functionals and forms, quadratic functionals and forms in real vector spaces
10. **Euclidean spaces:** inner product and the definition of a Euclidean space, Euclidean spaces as normed and metric spaces, orthonormal bases, isomorphisms and endomorphisms of Euclidean spaces, self-adjoint and orthogonal endomorphisms
11. **Elements of the tensor product for physicists**

Prerequisites and co-requisites

The student should know:

1. Basic operations on real and numbers, powers, and roots.

2. Properties of elementary functions.

Assessment methods and criteria

Subject passing criteria	Passing threshold	Percentage of the final grade
	51.0%	10.0%
	51.0%	40.0%
	51.0%	50.0%

Recommended reading

Basic literature

1. **A. Mostowski, M. Stark Algebra liniowa**, PWN, Warszawa 1976.

2. **W. Banaszak, W. Gajda Elementy algebry liniowej**, cz. I i II, WNT, Warszawa 2002.

3. **H. Arodź, K. Rościszewski Algebra i geometria analityczna w zadaniach**, Znak, Kraków 2005.

4. **Sheldon Axler Linear Algebra Done Right**, Springer, 4th ed., 2021.

5. **Gilbert Strang Introduction to Linear Algebra**, WellesleyCambridge Press, 5th ed., 2016.

6. **Steven Roman Advanced Linear Algebra**, Springer, 4th ed., 2023.

	Supplementary literature	• **David C. Lay, Steven R. Lay, Judi J. McDonald Linear Algebra and Its Applications**, Pearson, 5th ed., 2016. • **Howard Anton, Chris Rorres Elementary Linear Algebra with Applications**, Wiley, 12th ed., 2019. • **Jim Hefferon Linear Algebra**, 4th ed., 2023 .
	eResources addresses	
Example issues/ example questions/ tasks being completed		
Work placement	Not applicable	

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